An alternative model of International Migration: Endogenous Two Sided Borders and Optimal Legal Systems

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May 2, 2014

Abstract

In a 2-country and 3-period OLG model with education, we study the impact on international migration of the two sided characteristics of borders. Leaving a country does not mean entering the other one. Indeed, the country-specific social planner chooses the static welfare optimal level of education, of consumptions, of labor and of capital. There exists only one migration flow that is compatible with the market steady-state solution and these levels. Due to differences in education in each country, steady-state welfare maximizing levels of capital differ across countries. Consequently, both price differentials and incentives for illegal migration exist[1].

JEL Classification: K37, D91, F22
Key words: International Migration, Overlapping Generations Models, Immigration Law and Legal Systems.

[1]Remaining errors are ours. Corresponding author damien.gaumont@u-paris2.fr. We particularly thank Alice Mesnard for earlier discussion on illegal migration prior this paper. We also thank Bertrand Crettez and the participant of the City London University Seminar for helpful discussions on this paper. Authors are indebted to the participants of the OLG days in Clermont-Ferrand, May, 21st-23rd 2013, France, of the 62nd conference of AFSE, Aix-en-Provence, France as well as those of the International Conference PET13 (Public Economic Theory), Lisboa, Portugal. Special thanks to Antoine d'Autume, Boubtane Ekrame, Hyppolite d'Albis, Bertrand Wigniolle for their very helpful comments.

1 Introduction

Australia, Canada, the USA and New Zealand have all been set up by migration. However, nowadays, they have in common to implement migration programs to determine who is eligible to migrate. Australia and Canada have organized a legal system based on points to be accumulated by any would-be migrant. If they succeed to cross the threshold, would-be migrants are allowed to settle. A key feature of these legal systems of migration is that the threshold figure is not permanent. Quite to the contrary, it is optimally and legally set by governments given the specific economic needs of the country.

The objective of the paper is to provide a rationale to these optimal legal systems of international migration. Since countries adopt various migration criteria, among which education is an important one, this paper proposes a 3-period overlapping generations model in which individuals train when they are young and work when they are adults. Finally, when old, they optimally choose their retirement date. Such a model allows conclusions on both growth and welfare prior-migration (in autarky) and post-migration (when borders are open on their two sides).

The motivation for such a framework directly comes from empirical facts, which are analyzed through empirical as well as theoretical research. Withers (1987), for example, empirically shows that the skill level of migrant arriving in Australia has tended to increase in the postwar period at a more rapid rate than that of the resident population as a whole. In other words, the effectiveness of the points system in raising the mean skill level of immigrants depends on there being a large demand for visas to enter Australia. A study of the worldwide market for skilled immigrants by Cobb-Clark and Connolly (1997) suggests that the skills of those wanting to enter Australia are influenced by a range of factors, some of which are internal to Australia (e.g., economic conditions), while others are external (e.g., immigration policies of other countries). These factors are likely to have more impact on immigrant quality than the points system. The points system used in a number of the components of the immigration program in Australia offer a mean of selecting immigrants who will adjust rapidly to the circumstances of the Australian labor market and bring benefit to Australia. Variations in immigrant quality in Australia are likely to be affected more by conditions in the world-wide market for skilled immigrants than by the Australian points system. Understanding the worldwide market for skilled immigrants and determining the net benefits to Australia of different types of immigrants are important issues for consideration.

Theoretical literature concentrates on endogenous quotas of migrant through a voting system which allows government to implement immigration policies. Epstein and Nitzan (2005) analyze the endogenous determination of migration quota viewing it as an outcome of a two-stage political struggle between two interest groups: those in favor and those against the proposed migration quota. Theoretical effects of the government policy are depending on wether there are or not lobbying between those native who agree and those who disagree with the proposal of a quota of migrants. Mayrs (2010) derives a general equilibrium model with overlapping generations, where natives require a compensating wage differential for working in one sector rather than in another. Price and wage effects of immigration are analyzed on native: the young working in one of two sectors and the old. The outcome of a majority voting on immigration is determined into a given sector as well as the social optimum. The main findings are that i) the old determine any majority voting outcome of non-zero immigration into both sectors, ii) socially optimal immigration is smaller than or equal to the majority voting outcome, and iii) immigration is not necessarily a substitute for native mobility across sectors. Candau (2011) analyses how trade liberalization and immigration can potentially affect the welfare of native skilled and unskilled workers and how this expected impact plays on immigration policy. The novelty resides in the attempt to set up endogenous immigration restrictions by integrating swing voters in a model of geographical economics with two kind of immobile workers (skilled and unskilled). It is shown that trade liberalization can lead the winner candidate to increase quota on immigration.

Mayr (2012) determines occupation-specific immigration quotas in a political economy framework with endogenous prices and compares them to the social optimum. It shows that positive quotas for specific occupations can be the political outcome, even when total welfare effects of immigration are negative. Two of the main findings are that the (unique) voting outcome on immigration quotas is i) positive, if workers are immobile across occupations, and ii) negative (positive) for occupations where the native labor supply is sufficiently large (small), if workers are mobile across occupations.

Our model departs from the literature relative to endogenous quotas. Indeed, we propose an alternative way to obtain the optimal flow of migrants a country is willing to accept. The post migration steady-state equilibrium is a function of the flow of migrants, which is used as an instrument for the domestic migration policy. Since countries differ with respect to their return to education, incentives for migration exist. The way migration cease is not a pure market mechanism, but the result of the social planner's decision. The social planer chooses the level of migrants that lead his country to the post-migration static welfare optimum. There are no migration flow in the post-migration welfare steady-state equilibrium, even if each country chooses a different number of migrants compared with the other country. A natural consequence to this mechanism, contrary to Galor (1986), is that there are no price equalization in the long run.

The would-be migrants must be selected both by their home countries and the foreign countries. In that sense, we consider the two sided characteristics of any border. Most of the time the two-sided nature of border crossing is not theoretically analyzed, but empirically, legal migration systems take into account this double reality. In our model, even if countries have the same way of selecting migrants, they do not select the same level of migration flow. The emergence of the asymmetry of borders across countries is due to differences in the return to education. When one of the two countries elicits a higher return to education than the other, the flow of migrants optimally chosen by this country is not equal to the one chosen by the other country. In the case where one of the two country wants to send more migrants than the other one is ready to accept, incentive for illegal migration exists in post-migration equilibrium.

Section 2 presents the model, Section 3 the temporary equilibrium of the economy in autarky, Section 4 the inter-temporal equilibrium. Section 5 is devoted to international migration. Section 6 is devoted to economic interpretations of the theoretical results, Section 7 concludes.

2 THE MODEL

Consider a perfectly competitive international world with no uncertainty, with two countries, $i = 1, 2$, where economic activity in each country is operated over infinite discrete time, such that $t = 0, 1, 2, \ldots, \infty$. In every period, a new generation of individuals N_t^i is born and is supposed to be constant over time. Consequently, in autarky $N^i_{t+1} = N^i_t = N^i$ where $N^i = 1 > 0$ for simplicity. In each country, a single tradable good is produced using three factors of production: the capital, the adult efficient labor and the old efficient labor. As usual, capital depreciates fully after one period. Individuals and firms make rational decisions under perfect foresights.

2.1 THE INDIVIDUAL

Individuals are identical within as well as across generations. Individuals born in country $i = 1, 2$ live three periods, each of them being normalized to unity. In the first period when young, they borrow E^i_{t-1} on their future savings S^i_t when adult in order to train at the total cost ae^i_{t-1}, where a is the price of one unit of education e^i_{t-1} in country i. In the second period when adult, they supply ℓ^i_t subunits of labor, which is paid at the competitive wage w^i_t, so that the total earning of adult is $w^i_t\ell^i_t(e^i_{t-1})^{\varepsilon^i}$, where $0 < \varepsilon^i < 1$ is the country specific return to education. They consume c^i_t and the rest $S^i_t = s^i_t + R^i_t E^i_{t-1}$ is saved, where $R^i_t = 1 + r^i_t$ is the factor of interest and r^i_t the interest rate in country i during period t. The total saving S^i_t is devoted to s^i_t for the second period, and $R^i_t ae^i_{t-1}$ for reimbursing the first period training. In the third period when old, individuals consume d^i_{t+1} financed through the return on the second period saving $R^i_{t+1}s^i_t$, and their third period labor supply at wage p^i_{t+1} during θ^i_{t+1} subunits of time, where R^i_{t+1} and p^i_{t+1} are perfectly anticipated. Rational individuals maximize their log-linear utility function and solve the following program where β is the time preference and γ is the preference for leisure:

$$\max_{c^i_t, e^i_t, \ell^i_t, d^i_{t+1}, \theta^i_{t+1}} \log c^i_t + \gamma \log(1 - \ell^i_t) + \beta \log d^i_{t+1} + \beta\gamma \log(1 - \theta^i_{t+1})$$

subject to:

$$\left\{ \begin{array}{rcl} ae^i_{t-1} & = & E^i_{t-1}, \\ c^i_t + s^i_t + R^i_t ae^i_{t-1} & = & w^i_t \ell^i_t (e^i_{t-1})^{\varepsilon^i}, \\ d^i_{t+1} & = & R^i_{t+1} s^i_t + p^i_{t+1}\theta^i_{t+1}. \end{array} \right. \tag{1}$$

2.2 THE FIRM

In each country $i = 1, 2$, production occurs within a period according to a constant returns to scale production technology which is stationary over time. The output Q^i_t of the single goods is produced by a representative competitive firm at time t with three factors of production, capital K^i_t and young efficient labor $N^i_t = \ell^i_t(e^i_{t-1})^{\varepsilon^i}$ and old efficient labor $\Theta^i_t = N^i_{t-1}\theta^i_t = \theta^i_t$. The production technology is given by the following Cobb-Douglas production function $Q^i_t = K^{i\,1-\sigma-\nu}_t \left[\ell^i_t(e^i_{t-1})^{\varepsilon^i}\right]^\sigma \theta^{i\,\nu}_t$, where $0 < \sigma < 1$ is the elasticity of young efficient labor and $0 < \nu < 1$ is the elasticity of old efficient labor. The rational representative competitive firm maximizes its profit

$$\max_{K^i_t, \ell^i_t, \theta^i_t} \pi^i_t = K^{i\,1-\sigma-\nu}_t \left[\ell^i_t(e^i_{t-1})^{\varepsilon^i}\right]^\sigma \theta^{i\,\nu}_t - w^i_t\ell^i_t(e^i_{t-1})^{\varepsilon^i} - p^i_t\theta^i_t - R^i_t K^i_t. \tag{2}$$

We now turn to the study of the temporary equilibrium, which is the solution of the two previous problems, the one of the individual and the one of the firm.

3 Temporary equilibrium of the economy in autarky

The objective of this section is to determine the temporary equilibrium of the economy in autarky. For doing this, let us recall the definition.

Definition 1 *In country i, the temporary equilibrium of period t is a competitive equilibrium given perfect anticpations on prices, R^i_{t+1} and p^i_{t+1}, and given past variables, s^i_{t-1} and $I^i_{t-1} = N^i_{t-1}s^i_{t-1}$, or equivalently $K_t = s_{t-1}$.*

Lemma 1 *In temporary equilibrium, the adult efficient labor supply is constant, and the old efficient labor supply is constant too. We have $\ell^i_{t+1} = \ell^i_t = \ell^i$ and $\theta^i_{t+1} = \theta^i_t = \theta^i$.*

Proof. Consider the individual's problem 1. Solving the first period budget constraint for s^i_t and replacing its new expression into the second period budget constraint gives:

$$d^i_{t+1} = R^i_{t+1}\left[w^i_t\ell^i_t(e^i_{t-1})^{\varepsilon^i} - R^i_t a e^i_{t-1} - c^i_t\right] + p^i_{t+1}\theta^i_{t+1}. \tag{3}$$

Replacing (3) into the objective function, individuals solve the following program:

$$\max_{c^i_t, e^i_t, \ell^i_t, \theta^i_{t+1}} \log c^i_t + \gamma\log(1-\ell^i_t) + \beta\log\left[R^i_{t+1}\left[w^i_t\ell^i_t(e^i_{t-1})^{\varepsilon^i} - R^i_t a e^i_{t-1} - c^i_t\right] + p^i_{t+1}\theta^i_{t+1}\right]$$
$$+\beta\gamma\log(1-\theta^i_{t+1}).$$

The first order condition gives the following relations:

$$\frac{1}{c^i_t} = \frac{\beta R^i_{t+1}}{d^i_{t+1}}, \tag{4}$$

$$\varepsilon^i w^i_t\ell^i_t(e^i_{t-1})^{\varepsilon^i-1} = R^i_t a \iff (e^i_{t-1})^{\varepsilon^i} = \frac{R^i_t a e^i_{t-1}}{\varepsilon^i w^i_t \ell^i_t}, \tag{5}$$

$$\frac{\gamma}{1-\ell^i_t} = \frac{\beta R^i_{t+1} w^i_t (e^i_{t-1})^{\varepsilon^i}}{d^i_{t+1}}, \tag{6}$$

$$\frac{p^i_{t+1}}{d^i_{t+1}} = \frac{\gamma}{1-\theta^i_{t+1}}. \tag{7}$$

Rational competitive firm solves problem 2

$$\max_{K^i_t, \ell^i_t, \theta^i_t} \pi^i_t = {K^i_t}^{1-\sigma-\nu}\left[\ell^i_t(e^i_{t-1})^{\varepsilon^i}\right]^{\sigma}{\theta^i_t}^{\nu} - w^i_t\ell^i_t(e^i_{t-1})^{\varepsilon^i} - p^i_t\theta^i_t - R^i_t K^i_t.$$

The first order condition is:

$$(1-\sigma-\nu)Q^i_t = R^i_t K^i_t, \tag{8}$$

$$\sigma Q^i_t = w^i_t\ell^i_t(e^i_{t-1})^{\varepsilon^i}, \tag{9}$$

$$\nu Q_t^i = p_t^i \theta_t^i. \tag{10}$$

Using Definition 1 forward, $K_{t+1}^i = s_t^i$, rewrite the second period budget constraint forward as follows:

$$d_{t+1}^i = R_{t+1}^i K_{t+1}^i + p_{t+1}^i \theta_{t+1}^i$$

Using the first order condition of the firm (8) and (10)

$$d_{t+1}^i = (1-\sigma) Q_{t+1}^i, \tag{11}$$

that we put into (4) the first order condition of the individual to have

$$\frac{1}{c_t^i} = \beta \frac{(1-\sigma-\nu) Q_{t+1}^i}{(1-\sigma) Q_{t+1}^i K_{t+1}^i} \Longleftrightarrow c_t^i = \frac{(1-\sigma)}{\beta(1-\sigma-\nu)} K_{t+1}^i. \tag{12}$$

Put (12) into (6):

$$\frac{\gamma}{1-\ell_t^i} = \frac{\beta(1-\sigma-\nu)\sigma Q_t^i}{(1-\sigma) K_{t+1}^i \ell_t^i}. \tag{13}$$

By using (8) and (9), we have

$$\frac{\sigma Q_t^i}{K_{t+1}^i} = \frac{\beta(1-\sigma-\nu)+1-\sigma}{\beta(1-\sigma-\nu)(1-\varepsilon^i)}, \tag{14}$$

that we replace into (13) to have $\ell_{t+1}^i = \ell_t^i = \ell^i$ where

$$\ell^i = \frac{1-\sigma+\beta(1-\sigma-\nu)}{(1+\gamma(1-\varepsilon^i))(1-\sigma)+\beta(1-\sigma-\nu)}. \tag{15}$$

Note that using (4), we can rewrite (7) as

$$\frac{\beta \gamma c_t^i}{1-\theta_{t+1}^i} = \frac{P_{t+1}^i}{R_{t+1}^i}, \tag{16}$$

using (16) and (12), we have $\theta_{t+1}^i = \theta_t^i = \theta^i$ where

$$\theta^i = \frac{\nu}{\gamma(1-\sigma)+\nu}. \tag{17}$$

□

PROPERTY **1** *The old efficient labor supply is independent of the returns to education, ε^i, i.e. there is labor market integration of migrants when old.*

Using (5), (8) and (9), we have

$$e_{t-1}^i = \frac{\varepsilon^i \sigma K_t^i}{(1-\sigma-\nu)a} \quad \Longleftrightarrow \quad e_t^i = \frac{\varepsilon^i \sigma K_{t+1}^i}{(1-\sigma-\nu)a}. \tag{18}$$

PROPERTY **2** *The level of education is a linear function of capital and a decreasing function of the education cost, a, and of the returns to education.*

4 THE AUTARKIC PERFECT-FORESIGHT INTERTEMPORAL EQUILIBRIUM

The perfect-foresight inter-temporal equilibrium with constant population growth is obtained with the capital dynamics $K^i_{t+1} = s^i_t$.

LEMMA **2** *The dynamics of the economy is convergent*

$$K^i_{t+1} = \frac{\beta(1-\sigma-\nu)}{1-\sigma+\beta(1-\sigma-\nu)}\sigma(1-\varepsilon^i)\ell^{i\sigma}\left[\frac{\varepsilon^i\sigma}{(1-\sigma-\nu)a}\right]^{\varepsilon^i\sigma}\theta^{i\nu}K_t^{i\,1-(1-\varepsilon^i)\sigma-\nu}.$$

The steady-state equilibrium is unique

$$\overline{K}^i = \left[\frac{\beta(1-\sigma-\nu)\sigma(1-\varepsilon^i)}{1-\sigma+\beta(1-\sigma-\nu)}\left[\frac{\varepsilon^i\sigma}{a(1-\sigma-\nu)}\right]^{\varepsilon^i\sigma}\ell^{i\sigma}\theta^{i\nu}\right]^{\frac{1}{(1-\varepsilon^i)\sigma+\nu}}.$$

Proof. By Lemma 1, whatever the generation, efficient labors are constant over time so that the production of the current period t is

$$Q_t = K_t^{i\,1-\sigma-\nu}(\ell^i e^{i\ \varepsilon^i}_{t-1})^\sigma\theta^{i\nu}.$$

Using $K^i_{t+1} = s^i_t$ the dynamics of the economy is

$$K^i_{t+1} = w^i_t\ell^i_t(e^i_{t-1})^{\varepsilon^i} - R^i_t a e^i_{t-1} - c^i_t.$$

Using the first order condition of the firm (9) and the first order condition of the individuals (5) and (12), we have

$$\frac{1-\sigma+\beta(1-\sigma-\nu)}{\beta(1-\sigma-\nu)}K^i_{t+1} = \sigma(1-\varepsilon^i)Q^i_t,$$

$$K^i_{t+1} = \frac{\beta(1-\sigma-\nu)}{1-\sigma+\beta(1-\sigma-\nu)}\sigma(1-\varepsilon^i)Q^i_t.$$

Replace the production by its expression, we have

$$K^i_{t+1} = \frac{\beta(1-\sigma-\nu)}{1-\sigma+\beta(1-\sigma-\nu)}\sigma(1-\varepsilon^i)(\ell^i e^{i\ \varepsilon^i}_{t-1})^\sigma\theta^{i\nu}K_t^{i\,1-\sigma-\nu}.$$

Using (18) into $e^{i\ \varepsilon^i}_{t-1}$, we have:

$$K^i_{t+1} = \frac{\beta(1-\sigma-\nu)}{1-\sigma+\beta(1-\sigma-\nu)}\sigma(1-\varepsilon^i)\ell^{i\sigma}\left[\frac{\varepsilon^i\sigma K^i_t}{(1-\sigma-\nu)a}\right]^{\varepsilon^i\sigma}\theta^{i\nu}K_t^{i\,1-\sigma-\nu}.$$

Isolating K^i_t, the dynamics of the economy is convergent

$$K^i_{t+1} = \frac{\beta(1-\sigma-\nu)}{1-\sigma+\beta(1-\sigma-\nu)}\sigma(1-\varepsilon^i)\ell^{i\sigma}\left[\frac{\varepsilon^i\sigma}{(1-\sigma-\nu)a}\right]^{\varepsilon^i\sigma}\theta^{i\nu}K_t^{i\,1-(1-\varepsilon^i)\sigma-\nu}. \tag{19}$$

The steady-state equilibrium is unique

$$\overline{K}^i = \left[\frac{\beta(1-\sigma-\nu)\sigma(1-\varepsilon^i)}{(1-\sigma)+\beta(1-\sigma-\nu)}\left[\frac{\varepsilon^i\sigma}{a(1-\sigma-\nu)}\right]^{\varepsilon^i\sigma}\ell^{i\sigma}\theta^{i\nu}\right]^{\frac{1}{(1-\varepsilon^i)\sigma+\nu}}. \tag{20}$$

Note that the steady-state capital per worker is a quasi-concave function of ε^i. This will be important for the next Section. □

5 International Migration

Let us now consider that there are two countries, $i = 1, 2$. Countries are solely characterized by a difference in the return to education in the production function. We assume that the following inequality $\varepsilon^1 > \varepsilon^2$ holds for the rest of the paper. There are no other differences between countries. In country 2 the productivity of education is higher than in country 1, since $\varepsilon^i \in [0, 1]$.

5.1 Incentives for Permanent International Migration

Suppose that labor is permitted to migrate internationally. Let us assume that only young can permanently migrate. Migrants spend their education time, their working time as well as their leisure or their retirement time over the three periods in the immigration country. The borders between countries are supposed to be opened at time $t - 1 = 0$.

Proposition **1** *As long as* $\log\left[\frac{\varepsilon^2 e_1^1}{\varepsilon^1 e_1^2}\right] < \beta \log\left[\frac{Q_2^2}{Q_2^1}\right]$*, international migration is unilateral. Rational individuals born in country* i *have an incentive for permanent migration in country* j*, where* $i \neq j$*.*

Proof. Rational individuals born in country 1 have an incentive for permanent migration in country 2 if their indirect utility evaluated at the steady-state price system of country 2 over their life-cycle is higher than their indirect utility evaluated at the steady-state prices of country 1. The condition is:

$$\log c_1^1 + \gamma \log(1 - \ell_1^1) + \beta \log d_2^1 < \log c_1^2 + \gamma \log(1 - \ell_1^2) + \beta \log d_2^2.$$

Note that we know from the previous sections that the labor supply is an increasing function of the return of education, see (15), so that we have the following relationship

$$\gamma \log(1 - \ell_1^1) < \gamma \log(1 - \ell_1^2).$$

We now prove that

$$\log c_1^1 + \beta \log d_2^1 < \log c_1^2 + \beta \log d_2^2,$$

$$\log\left[\frac{c_1^1}{c_1^2}\right] < \beta \log\left[\frac{d_2^2}{d_2^1}\right].$$

Using relation (12)

$$c_1^i = \frac{1 - \sigma}{\beta(1 - \sigma - \nu)} K_2^i,$$

and using (18) we have

$$K_2^i = \frac{a(1 - \sigma - \nu)}{\sigma \varepsilon^i} e_1^i.$$

We have

$$c_1^i = \left[\frac{1 - \sigma}{\sigma}\right]\left[\frac{a}{\beta \varepsilon^i}\right] e_1^i.$$

Replace these expressions into the condition relative to the incentives for permanent migration

$$\log\left[\frac{\varepsilon^2 e_1^1}{\varepsilon^1 e_1^2}\right] < \beta \log\left[\frac{d_2^2}{d_2^1}\right].$$

Using relation (11) we have

$$\log\left[\frac{\varepsilon^2 e_1^1}{\varepsilon^1 e_1^2}\right] < \beta \log\left[\frac{Q_2^2}{Q_2^1}\right].$$

As long as $\varepsilon^1 e^2 > \varepsilon^2 e^1$, the left hand side is always negative, so that the condition is satisfied, considering that in the right hand side, the ratio of productions is greater than one[2]. □

5.2 DYNAMICS WITH PERMANENT INTERNATIONAL MIGRATION

Subsection 5.2 is devoted to the study of the dynamics of capital in country 2 and country 1. Without loss of generality, we will consider that incentive for migration are directed from country 1 to country 2. In such a situation, we only consider the case where only young are permitted to permanently migrate from country 1 to country 2. In steady-state equilibrium, period $t-1=0$, borders are open. A fraction m^i of the young is allowed to migrate. As it will be shown, m^i may be positive or negative, depending on the direction of the incentives for international migration. Consequently, according to the previous Subsection 5.1, $m^1 < 0$ characterizes the fact that individuals emigrate from country 1, while $m^2 > 0$ characterizes the fact that individuals immigrate in country 2.

Since after migration individuals are identical in each country — they train home if they do not migrate, or they train abroad if they migrate — in a given period $t \geq 2$, the population in country 2 is $L_t^2 = \ell_t^2 + m^2 \ell_t^1 = (1+m^2)\ell_t^2$ while the population in country 1 is $L_t^1 = (1-m^1)\ell_t^1$. Consequently, in each country efficient labor is defined as $L_t^2 e_{t-1}^{\varepsilon^2} = (1+m^2)\ell_t^2 e_{t-1}^{\varepsilon^2}$ and $L_t^1 e_{t-1}^{\varepsilon^1} = (1-m^1)\ell_t^1 e_{t-1}^{\varepsilon^1}$. The production function of country 2 is

$$\begin{aligned} Q_t^2 &= (K_t^2)^{1-\sigma-\nu}(1+m^2)^{\sigma}(\ell_t^2 e_{t-1}^{\varepsilon^2})^{\sigma}(1+m^2)^{\nu}\theta_t^{\nu} \\ \iff \quad Q_t^2 &= (1+m^2)^{\sigma+\nu}(K_t^2)^{1-\sigma-\nu}(l_t^2 e_{t-1}^{\varepsilon^2})^{\sigma}\theta_t^{\nu}. \end{aligned}$$

The production function of country 1 is

$$\begin{aligned} Q_t^1 &= (K_t^1)^{1-\sigma-\nu}(1-m^1)^{\sigma}(\ell_t^1 e_{t-1}^{\varepsilon^1})^{\sigma}(1-m^1)^{\nu}\theta_t^{\nu} \\ \iff \quad Q_t^1 &= (1-m^1)^{\sigma+\nu}(K_t^1)^{1-\sigma-\nu}(l_t^1 e_{t-1}^{\varepsilon^1})^{\sigma}\theta_t^{\nu}. \end{aligned}$$

Note there are no indexes on the old efficient labor, since whatever the country, old efficient labor supply is the same. Rational firm in country $i = 1, 2$ maximizes its profit,

$$\max_{K_t^2,\ell_t^2,\theta_t^2} (1+m^2)^{\sigma+\nu}(K_t^2)^{1-\sigma-\nu}(\ell_t^2 e_{t-1}^{\varepsilon^2})^{\sigma}\theta_t^{\nu} - w_t^2(1+m^2)\ell_t^2 e_{t-1}^{\varepsilon^2} - p_t^2(1+m^2)\theta_t^2 - R_t^2 K_t^2,$$

$$\max_{K_t^1,\ell_t^1,\theta_t^1} (1-m^1)^{\sigma+\nu}(K_t^1)^{1-\sigma-\nu}(\ell_t^1 e_{t-1}^{\varepsilon^1})^{\sigma}\theta_t^{\nu} - w_t^1(1-m^1)\ell_t^1 e_{t-1}^{\varepsilon^1} - p_t^1(1-m^1)\theta_t^1 - R_t^1 K_t^1.$$

[2]We can easily prove that such a situation exists. Indeed, suppose that $\varepsilon^1 > \varepsilon^2$ and that in the same time $\partial K/\partial\varepsilon^i < 0$ which occurs for high ε^2 since the steady-state capital per worker is a quasi concave function of ε^i. Using (18), the level of education e^i is a concave function of ε^i so that we have $e^2 > e^1$. Consequently $\varepsilon^1 e^2 > \varepsilon^2 e^1$ is satisfied. It is sufficient to note that the production is also a concave function of ε^i so that $\varepsilon^2 > \varepsilon^1$ is equivalent to $Q_2^2 > Q_2^1$, and the right hand side is positive. The inequality holds. One can also redo the same reasoning in the increasing part of the steady-state capital per worker by assuming $\varepsilon^2 > \varepsilon^1$ so that $Q_2^2 > Q_2^1$. Moreover, it exists many cases for which $\varepsilon^1 e^2 > \varepsilon^2 e^1$ is possible, especially when the difference in the return in education is high enough, $\varepsilon^2 - \varepsilon^1 > \alpha$ a positive number. Consequently, the same type of results arises in the increasing part of the steady-state capital per worker.

The first order condition for country $i = 1, 2$ where m^i is positive for $i = 2$ or negative for $i = 1$

$$(1-\sigma-\nu)\frac{Q_t^i}{1+m^i} = R_t^i \frac{K_t^i}{1+m^i}, \tag{21}$$

$$\sigma \frac{Q_t^i}{1+m^i} = w_t^i \ell_t^i e_{t-1}^{\varepsilon^i}, \tag{22}$$

$$\nu \frac{Q_t^i}{1+m^i} = p_t^i \theta_t^i. \tag{23}$$

Note that the following relations are unchanged compared with autarkic equilibrium, but now due to migration flows, the population can no longer be normalized to unity as it was the case in autarky. The dynamics of country 2 and country 1 are

$$K_{t+1}^2 = (1+m^2)s_t^2,$$
$$K_{t+1}^1 = (1-m^1)s_t^1.$$

Consequently, considering that $m^2 > 0$ and $m^1 < 0$, for country $i = 1, 2$ the individual's first period budget constraint and the individual's second period budget constraint are modified as follows.

$$\left\{ \begin{array}{rcl} ae_{t-1}^i &=& E_{t-1}^i, \\ c_t^i + \frac{k_{t+1}^i}{1+m^i} + R_t^i ae_{t-1}^i &=& w_t^i \ell_t^i (e_{t-1}^i)^{\varepsilon^i}, \\ d_{t+1}^i &=& R_{t+1}^i \frac{k_{t+1}^i}{1+m^i} + p_{t+1}^i \theta_{t+1}^i. \end{array} \right.$$

Using exactly the same procedure as in autarky, we obtain the new expressions of the consumption of the old is

$$d_{t+1}^i = (1-\sigma)\frac{Q_{t+1}^i}{1+m^i},$$

consumption of the young is

$$c_t^i = \left[\frac{1-\sigma}{\beta(1-\sigma-\nu)}\right] \frac{K_{t+1}^i}{1+m^i},$$

labor of the young and of the old are unchanged, and finally

$$e_{t-1}^i = \frac{\varepsilon^i \sigma}{(a(1-\sigma-\nu))} \frac{K_t^i}{1+m^i}.$$

Note that the young labor supply and the old labor supply and unchanged. Using the second period budget constraint, we can easily compute the steady-state capital per worker in each country.

$$\hat{K}^2 = \left[\frac{\beta(1-\sigma-\nu)\sigma(1-\varepsilon^i)(1+m^2)^{\nu+\sigma(1-\varepsilon^i)}}{(1-\sigma)+\beta(1-\sigma-\nu)} \left[\frac{\varepsilon^i \sigma}{a(1-\sigma-\nu)}\right]^{\varepsilon^i \sigma} \ell^{i\sigma} \theta^{i\nu}\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^i)}}, \tag{24}$$

$$\hat{K}^1 = \left[\frac{\beta(1-\sigma-\nu)\sigma(1-\varepsilon^i)(1-m^1)^{\nu+\sigma(1-\varepsilon^i)}}{(1-\sigma)+\beta(1-\sigma-\nu)} \left[\frac{\varepsilon^i \sigma}{a(1-\sigma-\nu)}\right]^{\varepsilon^i \sigma} \ell^{i\sigma} \theta^{i\nu}\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^i)}}. \tag{25}$$

Since both post-migration economies converge to a market steady-state equilibrium, we now investigate by which migration policy the social planer can guide the economy towards a first-best static welfare optimum. In standard overlapping generations models, this is designated as the Golden Rule and the government would calculate a tax system that leads the static per capita capital to maximize total consumption in that static state. Our problem is not exactly the same for two reasons. The first reason is that there is no tax system in our economy, and the second reason is that our problem is multidimensional. Since there is no tax system, the government uses the migration rate as a policy instrument in order to choose the static welfare maximizing level of education, adult and old labor and consumption, as well as the capital per worker ratio. Consequently, we must reformulate the social planer's problem, and this is the objective of the next Subsection.

5.3 The Static Welfare Optimum with permanent International Migration

We define the static welfare optimum of the economy and examine how it can be reached. It is defined as the stationary state that a social planner would select to maximize welfare under the feasibility constraint. The welfare criterion a collectivity must choose in order to rank all possible steady states has usually been described —following Samuelson (1958) — as the one that maximizes aggregate consumption. In standard models, this is called the Golden Rule and the government would calculate the static per capita capital that achieves this. Our problem is slightly different in the sense that now the social planner of each country $i = 1, 2$ maximizes the static welfare, and by doing this, he chooses the optimal levels of education e^i_w — where the subscript w captures the welfare maximizing solution of each variable —, adult labor ℓ^i_w and old labor θ^i_w, adult and old consumptions c^i_w and d^i_w, and the capital per worker k^i_w. He uses the level of migration m^i as an instrument to guide the economy toward the static welfare optimum, taking into account the macroeconomic equilibrium constraint of his country.

In the integrated world economy, the benevolent social planner in each country $i = 1, 2$ solves the following problem

$$\max_{K^i_w, \ell^i_w, \theta^i_w, e^i_w, c^i_w, d^i_w} \log[c^i_w] + \gamma \log(1 - \ell^i_w) + \beta \log[d^i_w] + \beta\gamma \log(1 - \theta^i_w),$$

subject to the macroeconomic equilibrium constraint

$$ae^i_w + c^i_w + d^i_w + K^i_w = {K^i_w}^{1-\sigma-\nu} (\ell^i_w {e^i_w}^{\varepsilon^i})^\sigma \theta^\nu_w.$$

In each country $i = 1, 2$ the first order condition is

$$(1 - \sigma - \nu) Q^i_w = K^i_w, \tag{26}$$

$$\frac{\sigma Q^i_w}{c^i_w \ell^i_w} = \frac{\gamma}{1 - \ell^i_w}, \tag{27}$$

$$ae^i_w = \varepsilon^i \sigma Q^i_w, \tag{28}$$

$$\frac{\nu Q^i_w}{c^i_w \theta^i} = \frac{\beta\gamma}{1 - \theta^i_w}, \tag{29}$$

$$d_w^i = \beta c_w^i. \tag{30}$$

The post migration macroeconomic constraint of the country 2 is as follows

$$c_w^i = Q_w^i - a e_w^i - d_w^i - K_w^i.$$

Using (26), (28) and (30) ad isolating $\frac{Q^i}{c^i}$ gives

$$\frac{Q_w^i}{c_w^i} = \frac{(1+\beta)}{\nu + (1-\varepsilon^i)\sigma}. \tag{31}$$

Put the last expression into (27) and isolating ℓ_w^i gives the optimal adult labor ℓ_w^i in each country $i = 1, 2$

$$\ell_w^i = \frac{\sigma(1+\beta)}{\sigma(1+\beta) + \gamma[\nu + \sigma(1-\varepsilon^i)]}. \tag{32}$$

Also, puting (31) into (29) and isolating θ_w^i gives the optimal old labor in each country $i = 1, 2$

$$\theta_w^i = \frac{\nu(1+\beta)}{\beta\gamma[\nu + \sigma(1-\varepsilon^i)] + \nu(1+\beta)}. \tag{33}$$

Using (26) into (28) and isolating e we find the expression of the chosen level in education in country i

$$e_w^i = \frac{\varepsilon^i \sigma K_w^i}{(1-\sigma-\nu)a}. \tag{34}$$

From relation (26) we deduce the optimal capital per worker that maximizes the welfare in each country

$$K_w^i = [(1-\sigma-\nu)(\frac{\varepsilon^i \sigma}{(1-\sigma-\nu)a})^{\varepsilon^i \sigma} \ell_w^\sigma \theta_w^\nu]^{\frac{1}{\nu+\sigma(1-\varepsilon^i)}}. \tag{35}$$

PROPOSITION **2**

1. *If the return of education is lower in country 2 than in country 1, the level of migration the social planer of country 2 implements is less than the one chosen by the social planer of country 1.*

2. *There are always incentives for illegal migration from country 1 toward country 2.*

Proof. To find the optimal level of migrants, we equalize $\hat{K}^i(m^i) = K_w^i$ so that $m^{i\star} = \Psi^{-1}(K_w^i)$. This lead to the expression of the welfare maximizing level of migrants for each country

$$m^{2\star} = \left[\left[\frac{1-\sigma+\beta(1-\sigma-\nu)}{\beta\sigma(1-\varepsilon^2)}\right] \frac{\ell^{2\sigma}_w}{\ell^{2\sigma}} \frac{\theta^{2\nu}_w}{\theta^{2\nu}}\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^2)}} - 1,$$

$$m^{1\star} = 1 - \left[\left[\frac{1-\sigma+\beta(1-\sigma-\nu)}{\beta\sigma(1-\varepsilon^1)}\right] \frac{\ell^{1\sigma}_w}{\ell^{1\sigma}} \frac{\theta^{1\nu}_w}{\theta^{1\nu}}\right]^{\frac{1}{\nu+\sigma(1-\varepsilon^1)}}.$$

□

LEMMA **3** *Since* $\ell^i{}_w, \ell^i, \theta^i_w$ *and* $1/\left[(1-\varepsilon^i)\theta^{i^\nu}\right]$ *are increasing functions of the return to education* $\varepsilon^i, i = 1, 2$. $m^{2\star}$ *is an increasing convex function of* ε^2 *and* $m^{1\star}$ *is a decreasing concave function of* ε^1.

Proof. The proof is given in Appendix 1. □

PROPOSITION **3** *There are incentives for illegal migration.*

Proof. For the incentive to migration to be from country 1 to country 2, it must be the case that $\mid m^{2\star} \mid < \mid m^{1\star} \mid$, and in the remaining of the paper we will assume this condition holds. If not, the direction of international migration is opposite. □

To sum up, the following graph illustrates the relative social planer's migration rates against the return to education in the production function of each country.

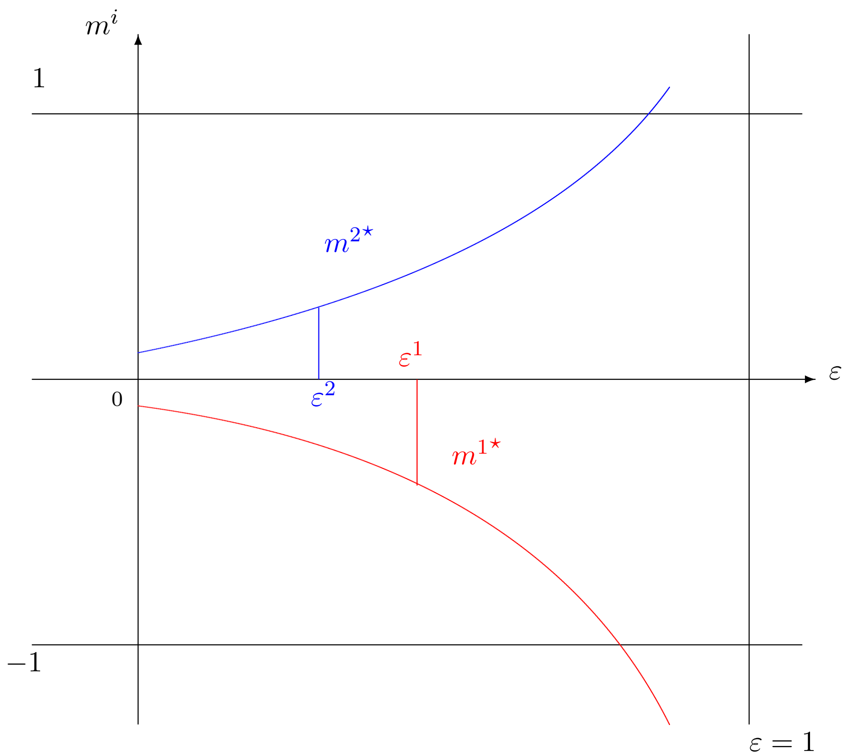

Figure 1: $\text{Elasticity}_{L_w/\varepsilon^2} + \text{Elasticity}_{\Theta_w/\varepsilon^2} \geq \Theta L \, \text{Elasticity}_{L/\varepsilon^2}$

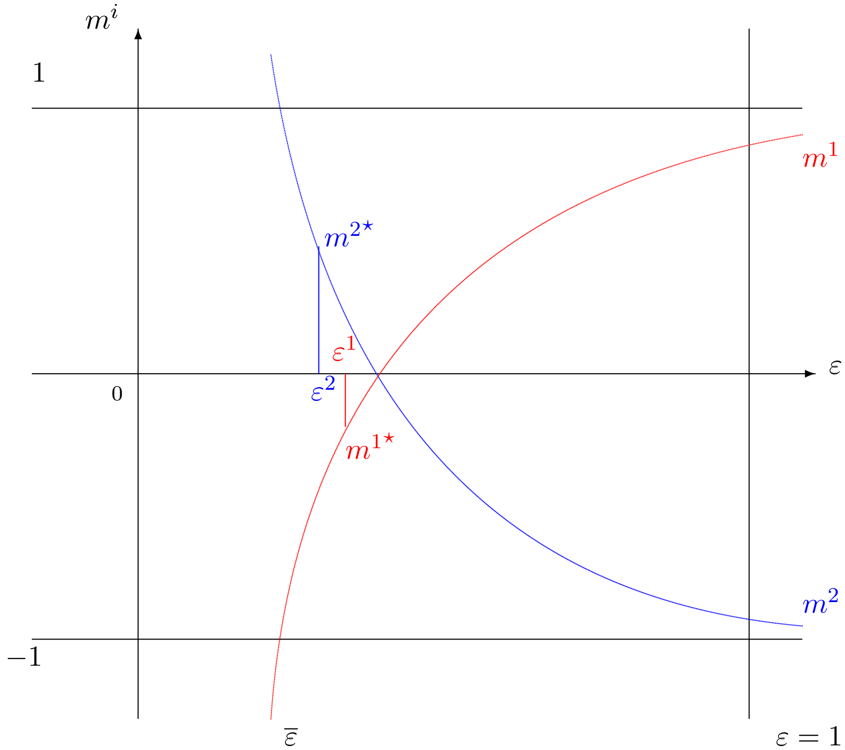

Figure 2: $\text{Elasticity}_{L_w/\varepsilon^2} + \text{Elasticity}_{\Theta_w/\varepsilon^2} < \Theta L\, \text{Elasticity}_{L/\varepsilon^2}$

In what follows, since we study post-migration perfect foresight equilibria, the post-migration flow is defined $m = \min\{m^1, m^2\}$ which is exactly anticipated by each country.

Bilateral migration flows may also emerge if the unilateral migration condition is not satisfied, and in that case, we have two possibilities

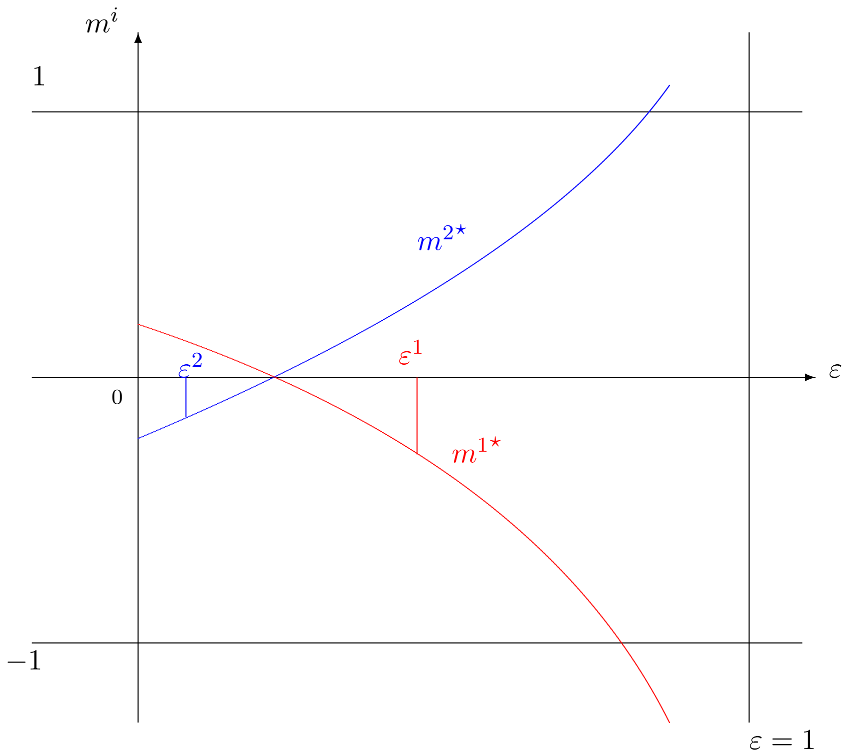

Figure 3: Bilateral Migrations: case 1

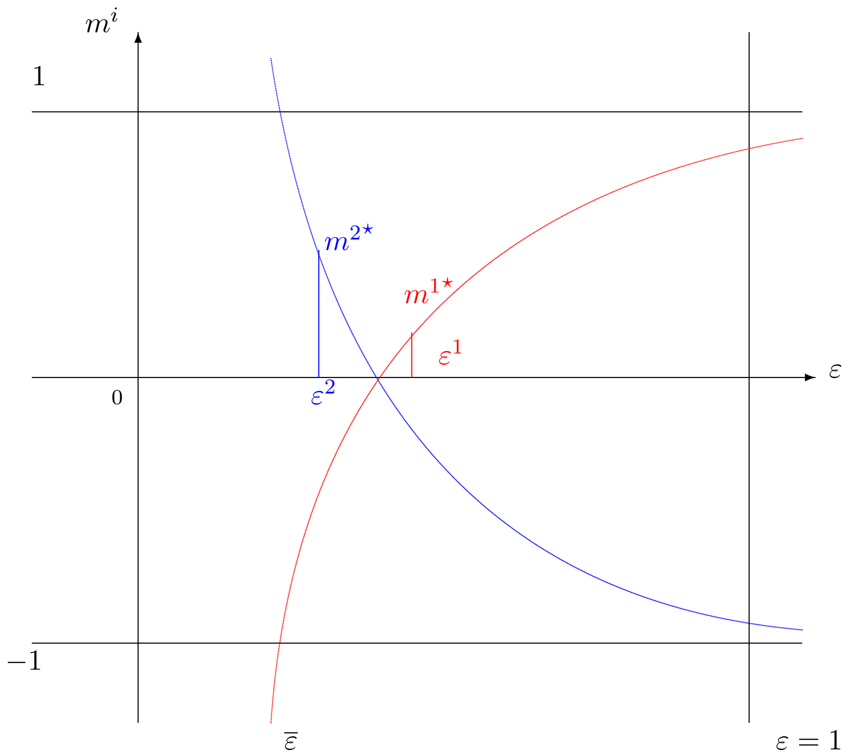

Figure 4:: Bilateral Migrations: case 2

5.4 Incentive for Illegal Migration

Each social planner maximizes the utility of his own country, consequently all education, consumption, labor and capital are set at their welfare maximizing level. When borders are open, there exists $m^{i\star}$ so that $\overline{K}^i(m^{i\star}) = K_w^i$ is satisfied.

Proposition **4** *In post-migration steady-state equilibrium, there are incentives for illegal migration.*

Proof. Let us consider the case were the optimal desired flows of migrants differ across countries, since $\varepsilon^2 < \varepsilon^1$. In that case and under the unilateral migration condition, the two migration flows satisfy the following inequality $\mid m^{1\star} \mid \geq \mid m^{2\star} \mid$. Consequently, there are incentives for country 1 to support illegal migration flows in direction of country 2[3].

5.5 The Emergence of an Optimal Price Differential Between Countries

Proposition **5** *In post-migration steady-state equilibrium, there are no prices equalization across countries.*

[3] An appropriate example is the wall between Mexico and the USA for which both countries were bargaining the number and places of each hole in the wall. Empirical literature includes case studies of Mexican communities that send illegal migrants to the United States, and estimates of the U.S. illegal - immigrant population, Hanson G.H. and A. Spilimbergo (1999). Frank D. Bean et al. (1990), using monthly INS data for 1977-1989, find that border apprehensions declined substantially following Immigration Reform and Control Act of 1986. Borjas et al. (1991), using annual INS data for 1967-1984, find that apprehensions by the U.S. Border Patrol are positively correlated with U.S. expenditure on border enforcement and U.S. real wages.

Proof. Since the returns to education differ across countries, the optimal migration policies lead the economies to different steady-state equilibria. Indeed, we have two main cases

1. First case is such that $m^{1\star} \geq m^{2\star}$, so that country 2 reaches the optimal level "before" country 1. In such a case, $\overline{K}^1(m^{1\star}) < K^1_w$ and $\overline{K}^2(m^{1\star}) = K^2_w$. Consequently, by assumption on the returns to education, $\varepsilon^1 > \varepsilon^2$ we necessarily have $\overline{K}^1(m^{1\star}) < \overline{K}^2(m^{1\star})$.

2. Second case is such that $m^{1\star} < m^{2\star}$, so that country 1 reaches the optimal level "before" country 2. In such a case, $\overline{K}^1(m^{1\star}) = K^1_w$ and $\overline{K}^2(m^{1\star}) > K^2_w$ according to our assumptions. Consequently, $\overline{K}^1(m^{1\star}) < \overline{K}^2(m^{1\star})$.

A natural consequence of such differences in steady-state capital is that there are no prices equalization across countries. It always remains a wage differential $\overline{w}^1 \neq \overline{w}^2$, and $\overline{p}^1 \neq \overline{p}^2$ as well as an interest rate differential across countries, $\overline{R}^1 \neq \overline{R}^2$. □

6 Economic Interpretations of the theoretical results

The focus of this theoretical paper is to provide a rationale for explaining how country-specific optimal legal systems emerge in order to regulate national migration flows. Double side borders have not been theoretically modeled in the literature. It is important to have theories taking into account that migration is a two step experience. Crossing borders means leaving one country (and cross the "exit" border) prior to enter the other one (and cross the "entrance" border). To our knowledge, this paper is a first attempt in that direction. The objective of this Section is to put our theoretical results in perspective with the existing legal system for both Canada and Australia. The relevance of the previous model is supported by empirical facts. Indeed, prior to migrate to Canada or to Australia, a migrant must apply for migration and if qualified, he/she can migrate. How does such a legal system works in practice?

6.1 The Canadian legal system of migration

The Canadian Visa of Immigration is obtained according to a legal system of points, see Chaabane 2011. The law sets how much points is it necessary to reach in order to be eligible to immigration. This number of points (67 points minimum over 100 possible in 2014) is flexible and changes depending on the economic needs of the country (73 points en 2004). The government can make migration easier or harder to obtain. The following conditions are required to be admissible:

1. to have a job offer,

2. to have been legal resident (Landing resident) for at least one year, or to have been a foreign student,

3. to be a qualified worker with at least one year of experience in one of the admissible industry of the country during the last 10 years.

Points are given according to various categories of criteria, which are public knowledge to any applicant to migration. Table 1 makes list of them.

Table 1: Migration Criteria and corresponding points

Criterium	Maximum Points
Education	25
Language (French or English)	24
Experience	21
Age < 49	10
Job offer	10
Adaptability	10
Total	100

The threshold is generally easier to reach for individuals who already are well educated. Note that applicants for migration are eligible if they have a sufficiently high level of savings already moved into a Canadian Bank. The required level of savings increases with the number of migrants, and is quite high for a family. It is interesting to underline that an applicant for migration should never have been prosecuted and must be in good health (a human capital condition). Table 2 presents this amounts for 2011.

Table 2: Necessary Saving Level for Migration to Canada

Persons	Saving in Canadian dollars
1	11 086
2	13 801
3	16 967
4	20 599
5	23 364
6	26 350
7	29 337

6.2 The Australian legal system of migration

Not only Canada has set up criteria for migration. Australia does too with the General Skilled Migration Program for individuals who are not sponsored by a "godfather" firm (or individual), but who are highly qualified in certain jobs for which there are specific Australian needs. Applicants should be aged by more than 18 years and not over pass 50 years in order to accumulate points. They must speak English, have an Australian Experience, especially in the "Australia's Skilled Occupation List" or have an Australian Diploma. Various Visa exists.

1. Onshore Visas are build for individuals who already are living in Australia and who want to be integrated in the General Skilled Migration Program.

2. The Offshore Visas are made for foreigners who apply for a permanent migration in Australia. This is the most important number of demands, and these Visas are restricted to qualified workers.

As for Canada, the number of points in 2011 was 65 points over 100 possible points. Those who do not reach the threshold enter a specific category called "reserve". If the number of points falls, they become immediately eligible prior any other current applicant.

6.3 The link between the existing legal system of migration and our results

As it seems clear, the first three criteria can be summed up into "education" in a wide sense, exactly as our theoretical approach does. That is the reason why we build a 3-period model with education. The social planner chooses these criteria in order to select migrants because those migrants will be economically useful for the country. In a broad sense, he maximizes the utility of native individuals (Canadian or Australian), especially if they bring more than the per capita saving. Such a criterion obviously favors Canadian or Australian growth of per capita capital, since a migrant must have a job to be eligible for migration. By doing so, the legal system of points — by determining the optimal migration flows — guides the economy in the direction of our theoretical concept of the static welfare optimum.

7 Conclusion

In a 3-period overlapping generations model with two countries, this paper proposed an alternative theory of international migration. Indeed, contrary to the traditional literature on international migration, in this model international migrations cease due to the optimal legal system each social planner implements in his country. Differences in social planer's decision are due to differences in the return to education across countries. As a consequence, each social planer does not chose the same level of migrants in each country, so that an optimal legal system for migration emerges, and generates endogenous two sided borders across countries. Even if each country uses the same way for designing its optimal international migration policy, the optimal level of migration flows varies across countries. A first natural consequence is the non equalization of prices, there always remains wage differentials and an interest rate differentials across countries in the post-migration steady-state equilibrium with optimal legal system of international migration. Since migration flows are unilateral, a second natural consequence of the non equalization of the steady-states is that incentive for illegal migration always exists.

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9 Appendix 1

In order to prove Lemma 3, one can derivate carefully the expressions $m^{i\star}$ in order to show that $m^{2\star}$ is an increasing convex function of ε^2 and $m^{1\star}$ is a decreasing concave function of ε^1.

Let us define $L_w = {\ell_w^2}^{\sigma}$, $L = \ell^{2\sigma}$, $\Theta_w = {\theta_w^2}^{\nu}$, $\Theta = \theta^{2\nu}$,

$$\frac{\partial m^{2\star}}{\partial \varepsilon^2} = \left[\frac{1-\sigma+\beta(1-\sigma-\nu)}{(\nu+\sigma(1-\varepsilon^2))\beta\sigma}\right]$$

$$\times \frac{\left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w\frac{\partial \Theta_w}{\partial \varepsilon^2}\right] L(1-\varepsilon^2)\Theta - L_w(1-\varepsilon^2)\Theta_w\left[\frac{\partial L}{\partial \varepsilon^2}\Theta + \frac{L\varepsilon^2\Theta}{(1-\varepsilon^2)^2\Theta^2}\right]}{L(1-\varepsilon^2)\Theta^2}$$

$$\times \left[\left[\frac{1-\sigma+\beta(1-\sigma-\nu)}{\beta\sigma(1-\varepsilon^2)}\right]\frac{L_w}{L}\frac{\Theta_w}{\Theta}\right]^{\frac{1-\nu-\sigma(1-\varepsilon^2)}{\nu+\sigma(1-\varepsilon^2)}}.$$

Note that the previous expression is positive if and only if the following condition is satisfied:

$$\forall \varepsilon^2 \neq 1, \left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w\frac{\partial \Theta_w}{\partial \varepsilon^2}\right] L \geq L_w\Theta_w\left[\frac{\partial L}{\partial \varepsilon^2} + \frac{L\varepsilon^2}{(1-\varepsilon^2)^2\Theta^2}\right].$$

The previous inequality is a condition relative to ε^2.

$$A(\varepsilon^2)^2 - B\varepsilon^2 + C \geq 0$$

where

$$A = \Theta\left[\left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w\frac{\partial \Theta_w}{\partial \varepsilon^2}\right]L - \Theta L_w \Theta_w \frac{\partial L}{\partial \varepsilon^2}\right],$$

$$B = \Theta\left[2\Theta^2 L_w \Theta_w \frac{\partial L}{\partial \varepsilon^2} - L_w \Theta_w L - 2\Theta L\left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w\frac{\partial \Theta_w}{\partial \varepsilon^2}\right]\right],$$

$$C = \Theta\left[\frac{\partial L_w}{\partial \varepsilon^2}\Theta_w + L_w\frac{\partial \Theta_w}{\partial \varepsilon^2}\right]L - \Theta^2 L_w \Theta_w \frac{\partial L}{\partial \varepsilon^2}.$$

As long as we have $A > 0$ the migration flow is an increasing convex function of ε^2 which is equivalent to the following condition, which holds for reasonable values of parameters.

$$\text{Elasticity}_{L_w/\varepsilon^2} + \text{Elasticity}_{\Theta_w/\varepsilon^2} \geq \Theta L\, \text{Elasticity}_{L/\varepsilon^2}$$

We have shown that m^2 is an increasing and convex migration function. Consequently, for country 2, the lower the return of education ε^2 the higher the migration flows. In the same way, m^1 is a decreasing concave function of the return to education.